Vol. 10, No. 4, 2024, pp. 349-357
DOI: https://doi.org/10.29210/020244627

Contents lists available at Journal IICET

JPPI (Indonesian Journal of Educational Research)
ISSN: 2502-8103 (Print) ISSN: 2477-8524 (Electronic)

Journal homepage: https://jurnal.iicet.org/index.php/jppi

Making sense of fraction in elementary on montessori didactic materials

Rosalin Rosalin*), Tatang Tatang
Pascasarjana Universitas Pendidikan Indonesia, Bandung , Indonesia

Article Info

Article history:

Received Sept 19th, 2024
Revised Oct 20th, 2024
Accepted Nov 25th, 2024

Keyword:

Fractions
Manipulative
Montessori didactic materials

ABSTRAK

This study aims to explore how Montessori didactic materials help elementary school students understand the concept of fractions. Using qualitative methods and a Design Didactic Research (DDR) approach, data were collected through classroom observations, semi-structured interviews with Montessori teachers, and documentation of student work. The data analysis technique used was thematic analysis, where the researcher identified patterns and main themes from the data collected. Triangulation was conducted to validate the research results by combining findings from interviews, observations, and documentation. The results showed that the use of concrete manipulative materials significantly improved students' understanding of fractions, developed problem-solving skills, and built a strong connection between abstract and real concepts. The teacher acts as a facilitator, allowing students to learn independently while still guiding them in the exploration process. This study concludes that the Montessori method is effective in teaching fractions at the primary school level, by providing a deep and meaningful learning experience for students.

Corresponding Author:

Rosalin Rosalin,
Universitas Pendidikan Indonesia
Email: rosalin.devi21@upi.edu

Introduction

The teaching of fraction concepts in primary school has a very important role because understanding fractions is a basic skill that underlies many further mathematical concepts, such as algebra, geometry, and even calculus (Anggreini & Priyojadmiko, 2022); (Wilkie & Roche, 2023). Fractions not only help students understand division and proportion, but also teach them how to work with parts of a whole which is an important basis in many mathematical contexts and everyday life, such as measurement and time or money management (Yayuk, 2019). By understanding fractions early on, students will be better prepared to face mathematical challenges in the next level of education, as well as develop logical and analytical thinking skills (Rosanah, 2022); (Kofa, 2017); (Crneckiy, 2022).

Many students face difficulties in understanding the concept of fractions due to their abstract nature, in contrast to whole numbers which are more concrete and familiar (Ummah, 2021). Fractions involve dividing a whole into smaller parts, which is not always easily understood through traditional methods such as lectures or textbooks (Monzon, 2020). This difficulty often arises because fractions demand an understanding of the relationship between parts and wholes, as well as the concepts of comparison and proportion, which can be confusing without visualisation or concrete aids. In addition, mathematical operations on fractions, such as

addition or subtraction with different denominators, often confuse students if they have not deeply understood the basics of fractions (Alshehri, 2017); (Beilstein, 2019). Therefore, more interactive learning approaches, such as the use of manipulatives or practical activities, are often more effective in helping students understand this concept.

The Montessori method offers a highly effective approach to introduce abstract concepts such as fractions to children in a more concrete and visual way (Cipta et al., 2020); (Tamara, 2022). This approach focuses on hands-on learning and the use of didactic materials, such as fraction blocks, discs, and props specifically designed to visually explain fraction concepts (Pouw, 2017) . By manipulating these materials, students can see and feel how the parts make up the whole, as well as understand the relationship between different fractions more clearly. The Montessori method encourages independent exploration and experiential learning so that students can internalise fraction concepts through practical activities that make abstract mathematical ideas more concrete and easily understood (Goss, 2022); (Okoth, 2021).

Previous research related to the effectiveness of using the Montessori method shows that this approach can significantly improve students' mathematical understanding and skills compared to traditional teaching methods. Several studies have identified that Montessori didactic materials, such as fraction circles and fraction insets, help students understand abstract concepts in a more concrete and visual way. Research by (Aziza et al., 2020) showed that students taught using the Montessori method showed better development in basic maths skills and problem solving compared to students who followed the traditional curriculum. In addition, a study by (Hanifah et al., 2024) showed that the Montessori learning method has an effective impact on the development of attention in ADHD children. The association of ADHD children's attention through this method intervention lies in the main characteristics of Montessori, namely a structured environment and freedom within certain limits, where children can choose activities of interest, thus extending the duration of their concentration. Activity-based Montessori interventions using sensory materials make children more engaged, concentrated, and actively participate in the learning process. These findings support the premise that the Montessori approach can provide advantages in teaching mathematics, especially in understanding complex concepts such as fractions.

This research is very important in the current learning context as it can provide valuable insights into the effectiveness of using Montessori materials in improving students' understanding of fractions compared to traditional teaching methods. By examining the Montessori approach that emphasises hands-on learning and the use of concrete didactic materials, this study can show the extent to which this method is more effective in helping students understand abstract fraction concepts. The results of this study can offer empirical evidence supporting or rejecting the use of the Montessori method in the mathematics curriculum, as well as provide valuable information for educators to design more efficient teaching strategies that suit students' needs. In addition, this study can broaden the understanding of how visual and manipulative approaches can improve basic maths skills in the classroom and contribute to better educational practices.

The purpose of this study is to understand how Montessori didactic materials are used effectively by teachers in helping primary school students understand the concept of fractions. The study also aimed to explore teachers' experiences in teaching fractions using concrete manipulatives, as well as assess the extent to which the materials help students in building conceptual understanding of fractions. In addition, this study wants to analyse how the interaction between students, teachers, and didactic materials affect the learning process of fractions, as well as identify the challenges teachers face in their implementation in Montessori classrooms.

Method

This study uses a qualitative research method with a Design Didactic Research (DDR) approach. This research aims to explore how Montessori didactic materials are used by teachers to help primary school students understand the concept of fractions. DDR enables an extensive analysis of a particular phenomenon within its authentic context of learning obstacles (Fauzi & Arini, 2021; Muhamad G Isnawan et al., 2022; Suryadi, 2019). This study used a combination of purposive sampling and convenience sampling techniques to obtain comprehensive data related to the use of concrete manipulatives in teaching. Purposive sampling was used to select two Montessori teachers with more than 15 years of experience, aiming to explore the effectiveness of concrete manipulatives in teaching fractions as well as the role of teachers in guiding students' understanding individually. On the other hand, convenience sampling was applied to select students from vertical classes (grades 1 to 3) based on ease of access, to investigate how the manipulatives are used in the learning of students of different age levels. The combination of these two techniques provided an in-depth view from the experienced teacher's side as well as representative data from students, by minimizing potential bias through class representativeness and variation in students' responses to concrete manipulatives.

This study was conducted at a Montessori Elementary School in Bandung in grades 1 to 3 with a total of 31 students, consisting of 14 students in grade 1, 10 students in grade 2, and 9 students in grade 3. Observations and interviews took place in April 2023, towards the end of the school year. The other participants were two teachers (Teacher A and Teacher Y) who were interviewed to obtain information about students' understanding of fractions. The Montessori classroom uses a vertical learning structure with a mixed age group (6-9 years old), where fraction learning includes basic concepts, addition, and subtraction of fractions. The main data collection method was semi-structured interviews with two certified Montessori teachers who have been teaching for more than 15 years. The interviews aimed to explore their experiences in teaching maths and the use of concrete manipulatives to aid student learning. Documentation from teacher observation sheets was also collected to support the findings, along with simple worksheets on fractions for students, as well as documentation of students learning fractions. Data were obtained through classroom observations, interviews and documentation of students' work.

The data collected connects teachers‘ teaching methods with students’ use of didactic materials in understanding fraction concepts. Didactic materials in the Montessori environment in the form of practical activities, teaching aids, interviews, classroom observations, student journals, and documentation were used to analyse the effectiveness of these materials in learning fractions and their characteristics. The author applied the triangulation technique to increase the validity of the results by combining data from interviews, observations, and documentation. Interviews with experienced teachers lasted 75 to 110 minutes, with an average interview duration of 90 minutes. This triangulation ensured that data was verified through multiple methods, providing an in-depth picture of the use of Montessori manipulatives in understanding fractions. Interviews provided direct insight into the teaching experience, while classroom observations allowed the researcher to see students' use of manipulatives. Documentation such as teacher observation sheets and student work served as additional evidence supporting the findings from interviews and observations. By combining these three methods, the researcher can minimize bias and increase the accuracy and credibility of the research results.

The Design Didactic Research (DDR) stage in this study began with the development of a research design that focused on identifying students' learning problems in understanding fraction concepts. The researcher developed a DDR framework to analyse how teachers use Montessori didactic materials in learning fractions. Data collection was conducted through classroom observations, semi-structured interviews with teachers, as well as documentation from observation sheets and students' review papers, which aimed to observe the use of concrete manipulatives by students and the role of teachers in facilitating such learning. Furthermore, data analysis was conducted qualitatively by identifying patterns and relationships through triangulation of data from various sources. The analysis process involved coding the results of interviews and observations to find the main themes related to students' understanding of fractions. A research flowchart can be used to visualise the stages of this DDR more systematically.

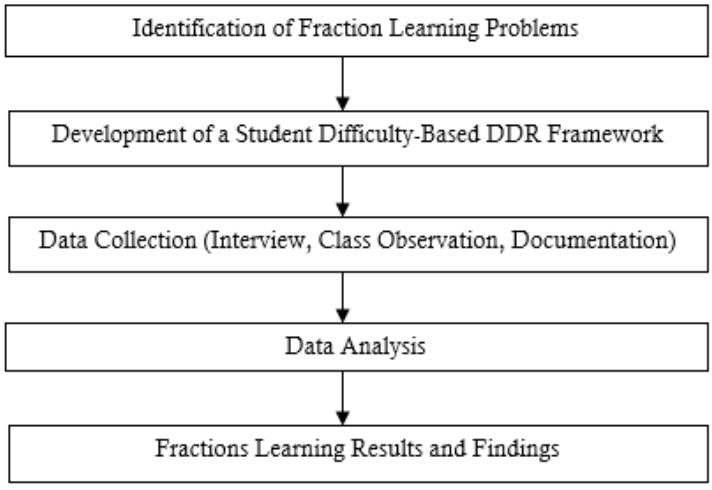


Figure 1 <Flowchart of the Design Didactic Research (DDR) Stages>

Results and Discussion

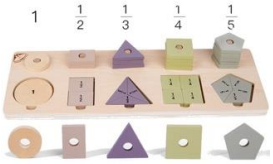
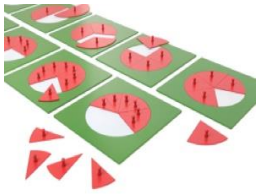
The Use of Montessori Didactic Materials for Fraction Learning

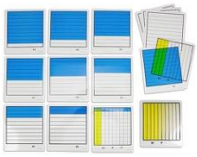
Dr. Maria Montessori established education method under her name more than century ago. Her education method focused on child-centered approach, which is grounded in scientific observation of human from infancy

to adulthood (Culclasure et al., 2019). The key elements of establishing a genuine Montessori program are the allocation of uninterrupted blocks of work time, stimulation of independent work and the provision of guided choices for work activities. Didactic materials in Montessori setting. Didactic material in Montessori environment promotes the growth of senses, facilitate the comprehend of abstract concepts in tangible ways, and fortify the connection between brain and hands (Phillips, 2022). While (M. G. Isnawan, 2023) postulated didactic design in learning is to prepare strategy of giving lesson based on the analysis of students' learning obstacles and difficulties in learning. Maria Montessori required that teachers must be well-trained to master didactic materials to attain competencies in broaden experience for better education (Ahlquist & Gynther, 2020).

Montessori fractions materials are designed to provide students with hands-on experience and visual representations of fractions. These materials help students to develop a concrete understanding of mathematical concept of fractions. Fraction Skittle is the first Montessori fraction material to introduce fractions to introduce in lower elementary. The Skittles have its own range of pictorial and abstract symbols. This early fraction models are tangible and easy to understand by young students. They can manipulate and relate to pictures and their corresponding symbols. Exploring didactic materials enhance their understanding of a part whole in abstract symbols (Ahlquist & Gynther, 2020); (Asoy et al., 2022); (Parungao, 2021). The purpose of combining of concrete and symbols in fraction is to make connection between both representations. These activities construct mental mathematical skill that is important for the preparation of abstract thinking (Parungao, 2021). Students who frequently work with didactic materials are more confident and independent in doing their tasks.

The hallmark of Montessori fraction materials is didactic. They are precisely crafted and emphasis in sensory experience comply with developmentally appropriate progression. The pedagogical approach of materials is meant to meet the needs of students in learning. The qualities of Montessori materials are: (1) Hands-on and interactive. (2) Self-correcting with control of error. (3) Sequential and order. (4) Concrete and tactile. (5) Open ended and versatile. (6) Multisensory approach to help student learns in holistic way.(7) Beautiful and inviting. The Various materials shown in *Table 1* are precisely designed and esthetically pleasing. Each of sets isolate one quality of knowledge with control of error(s) allow children to make self-correction and work independently. The didactic materials are also supporting collaborative group work that are effective in improving students' learning ability and reasoning skills (Hallumoğlu et al., 2023).

Name of Materials	Picture	Name of Materials	Picture
Fraction Skittles		Fractions Shapes	
Fraction Linear		Fraction Circles	
Fraction Squares		Fraction Triangles	
Cut-out Didactic Fraction Circles 1 – 1/10		Cut -out Didactical Fraction Circle 1/11 – 1/20	

Fraction Circle for Learning Degree		Decimal Fractions Set	
Theorema of Phytagoras		Fraction Transparency	

Montessori elementary classroom, various concrete materials to learn fractions are provided in accommodating different learning style of students to build foundation of fractions. The didactic material in Montessori mathematics are geometric shapes allows students multisensory experience and internalize concepts follow their pace and style of learning mathematics (Ahlquist & Gynther, 2020; Culclasure et al., 2019).Whilst this study focuses on optimizing fractions, mathematical concrete tools will be explored to enhance understanding of fractions. Didactical Montessori materials.

The Correlation Between the Montessori Approach and Understanding of Fraction Concepts

The child-centered Montessori approach to learning fractions provides students with the opportunity to understand the concept more deeply compared to conventional teaching methods. In the Montessori system, students learn through hands-on experiences using concrete manipulatives specifically designed to illustrate mathematical abstractions. For example, in learning fractions, students use fraction circles that can be touched, split, and reassembled, allowing them to physically grasp how fractions work. Through this approach, students do not merely memorize formulas but genuinely understand the meaning behind fraction operations, such as division and subtraction, which are presented visually and kinesthetically. This contrasts with conventional methods that often rely on abstract symbols and verbal explanations without direct student involvement.

In an interview with an experienced Montessori teacher, Teacher A explained, "Children find it easier to understand fractions when they can see and touch the parts. When I give them fraction circles and ask them to divide them, they naturally begin to grasp that half of something is two equal parts. This is more effective than just explaining it verbally." This statement highlights that students' direct interaction with learning materials significantly aids in internalizing the concept of fractions. Teacher A also added that this approach boosts students' confidence as they feel they can "control" the concept being studied, rather than just memorizing facts or rules. Classroom observations showed that students were highly engaged in learning fractions when using manipulative materials. Younger students, for example, were enthusiastic when comparing the sizes of various fractions, such as halves, thirds, and quarters, by dividing circles and testing their fit. "It's amazing to see how students experiment with fractions without the fear of making mistakes," said Teacher Y. The students' responses, displaying enthusiasm and curiosity, indicate that the Montessori approach not only enhances their conceptual understanding but also fosters curiosity and engagement in learning.

Compared to conventional teaching methods, where students might only be given fraction problems to solve symbolically, the Montessori approach offers a more holistic experience. Conventional methods tend to rely on memorization and practice problems, which can lead students to miss the true meaning of the concepts they are learning. In contrast, Montessori encourages students to learn through experimentation and exploration, fostering a deeper and more lasting understanding. The positive responses from students, as observed in the classroom, reinforce the argument that experiential learning in the Montessori approach yields more meaningful results in understanding mathematical concepts, especially fractions.

Related literature shows that the Montessori method significantly enhances students' ability to grasp abstract concepts such as fractions. A study by (Lillard, 2017) revealed that concrete didactic materials in the Montessori approach, such as fraction insets and fraction skittles, allow students to visualize and manipulate fraction concepts directly, helping to reinforce their understanding. Another study by (Hamsa, 2020) emphasized that student independence in Montessori provides space for them to explore mathematical concepts at their own pace. This aligns with (Rosanah, 2022) findings, which showed that exploration-based learning, as applied in Montessori, can improve problem-solving skills and students' long-term understanding of the relationships

between numbers in fractions. This literature supports the argument that the Montessori approach is more effective in developing a stronger conceptual understanding of fractions compared to conventional methods.

The Impact of Montessori on Long-Term Fraction Understanding

The Montessori approach has a significant impact on long-term understanding of fractions. By using concrete materials like fraction circles and bars, students engage with fractions in a hands-on manner, which allows them to visualize and internalize abstract mathematical concepts. This method enhances not only their comprehension of fractions but also their ability to apply this knowledge in various problem-solving scenarios. Unlike traditional methods, which often focus on memorization, Montessori encourages deeper exploration, helping students build a strong foundation in mathematical reasoning that can be retained over time. As a result, students who are taught fractions through Montessori materials tend to develop a more intuitive grasp of the relationships between numbers and fractions. The data in the table, which highlights the frequency of students' understanding of concepts like fraction addition and subtraction, further illustrates the effectiveness of the Montessori method**.**

Table 1 <The Understanding Fraction Concept>

		Fraction Concept			
		Frequency	**Percent**	**Valid Percent**	**Cumulative Percent**
Valid	Yes	31	96.9	96.9	96.9
	Yess	1	3.1	3.1	100.0
	Total	32	100.0	100.0	

Based on the data presented in Table 2, it can be concluded that the understanding of the fraction concept has been mastered by almost all students. This can be attributed to the use of concrete representations as learning aids, which aim to enhance students' mathematical skills. This method has been implemented since of first grade, allowing students to be introduced to the concept of fractions in an easily comprehensible context from an early stage. With this approach, students not only learn the theory but also gain practical experience that reinforces their understanding of fractions. The consistent and sustained implementation of this method proves effective in ensuring that nearly all students can master the concept of fractions effectively.

Tabel 2 <The understanding of fraction Addition>

		Fraction Concept			
		Frequency	**Percent**	**Valid Percent**	**Cumulative Percent**
Valid	Yes	17	53.1	53.1	53.1
	Struggle	1	3.1	3.1	56.3
	Not Yet Given	14	43.8	43.8	100.0
	Total	32	100.0	100.0	

Based on Table 3, 53.1% of the students who learned fraction addition were able to perform the operation independently. Fraction addition was introduced at the end of first grade using concrete representations coupled with images representing their concrete objects to support the period of abstraction. This approach allows students to more easily grasp the concept of fractions. Teachers also encourage student collaboration in working with fractions to broaden their experience and understanding. However, one student experienced difficulties with adding and subtracting fractions. According to the teacher, this student lacked proficiency in using concrete fraction materials, which became a barrier in the learning process.

Table 3 <Understanding Fraction Subtraction>

		Fraction Concept			
		Frequency	**Percent**	**Valid Percent**	**Cumulative Percent**
Valid	Yes	8	25.0	25.0	25.0
	Struggle	1	3.1	3.1	28.1
	Not Yet Given	23	71.9	71.9	100.0
	Total	32	100.0	100.0	

The result of the data is in Table 4 is strictly discuss about fractions substraction with same denominators. Eight of nine students of grade 3 can do simple fraction subtraction, meaning that concrete representation helped student understanding fraction subtraction effectively. One student needed more guidance with fraction concepts and operations in the third grade. The class teacher identified these challenges caused by the student's insufficient familiarity with physical manipulatives compared to his peers. However, the number of students who understood fraction concepts and could do fractions operations increased. This consistent rise in third-grade students demonstrating an understanding of fraction operations is noteworthy in Montessori classrooms.

Exposure to more concrete models, such as long, thin rectangles, number lines, circles, and linear grids or dot paper, has enhanced students' mastery of mathematics (Bungao-Abarquez, 2020); (Culclasure et al., 2019). These models also provide visual representations facilitating understanding, problem-solving, self-efficacy, and mathematical reasoning.

The Role of the Teacher in the Montessori Approach

The role of the teacher in learning fractions through Montessori materials is very important, although this approach emphasizes student independence. The teacher functions as a facilitator who guides students to independently discover and understand the concept of fractions through the exploration of manipulative materials such as fraction circles or number rods. In the Montessori method, the teacher does not give lengthy explanations or directly tell the answers, but rather gives brief initial instructions before letting students do their own exploration. The teacher will observe and record the progress of each student, then provide additional guidance if needed, for example by asking questions that provoke students to think more deeply about the relationship between the parts of the fraction and the whole.

Teachers' views on their role in this process are very positive, as they see students as more active in constructing their own understanding. An experienced Montessori teacher stated, "I focus more on listening to the students' questions and seeing how they try to find the answers themselves with the materials at hand. I only give direction when they really need help." This approach allows students to feel that they have control over their learning process, which in turn increases their confidence and long-term understanding of fraction concepts. In assessing student progress, Montessori teachers do not use standardized written tests, but rather more holistic observation-based assessments. They look at how students utilize Montessori materials, such as whether students can fluently combine and separate fractions, as well as understand the relationship between fractional numbers. In addition, teachers also observe the development of students' problem-solving skills, for example when they are asked to find ways to combine fractions to reach a whole number. The teacher also records this progress in an observation sheet that documents each step and the understanding achieved by the students.

The strategies teachers use to support understanding of fractions involve giving students the opportunity to repeat explorations with manipulative materials as often as possible, without strict time limits. Teachers also integrate small group discussions to help students share their findings on fractions. In these groups, students are encouraged to explain to each other how they understand the concept of fractions through the materials they use, which strengthens their own understanding and helps other students who may still be confused. This approach reflects the teacher's role not only as a knowledge provider, but also as a guide who supports students' individual cognitive development.

In the literature related to practice-based learning and the role of teachers in the Montessori approach, various studies show that this approach encourages student independence while placing the teacher as a facilitator. (Meiana, 2023); (Utami, 2023) stated that in Montessori education, the teacher's role is not only as a teacher, but more as an observer and guide, who directs students through independent exploration with manipulative materials. Another study by (Vivant, 2022) revealed that practice-based learning through concrete materials allows students to develop a deeper and more intuitive understanding of abstract concepts, such as fractions. Teachers in the Montessori approach play an important role in creating an environment that supports students' independent exploration and interaction with teaching materials, while observing their progress individually and providing appropriate guidance when needed (Azizah, 2024); (Saylor et al., 2018);(Mavric, 2020).

Challenges in Implementing Montessori Materials for Fraction Concepts

In the context of Montessori primary schools, teaching using Montessori didactic materials is common practice, but there are challenges in implementing fraction teaching. One possible challenge is the lack of individualisation in a class with a larger number of students. Although the Montessori method focuses on independent exploration, teachers still need to ensure that each student gets individualised guidance, which can be challenging if students have different levels of understanding. In a class with many students, teachers need to effectively monitor each child, which sometimes requires more time and attention.

In addition, although students in Montessori elementary schools are familiar with the use of manipulative materials, not all students may immediately be able to grasp abstract concepts such as fractions just by independent exploration. There are students who need more intensive instruction or additional intervention from the teacher. This is where the teacher's role as a facilitator is crucial. Another challenge is varying the didactic materials to suit different students' learning styles, especially if there are students who have difficulty transitioning from concrete manipulatives to more abstract fraction concepts. Another challenge is the time it takes to achieve deep understanding. Learning with the Montessori method often takes longer than with conventional methods. Students may need more time to explore various materials before really understanding

the concept of fractions in depth. For teachers, this can be a challenge if the curriculum to be completed has a specific deadline. Managing time while still providing opportunities for students to learn independently can affect the pace and effectiveness of learning.

In some previous studies, the challenges in using Montessori didactic materials, especially in teaching fraction concepts, are often related to students' limited adaptation to the use of concrete tools and abstraction of mathematical concepts. According to a study conducted by (Lillard, 2017), one of the main challenges faced is the difference in students' ability to move from physical manipulation to abstract understanding, especially when students have to apply the concept outside the context of Montessori tools. In addition, other literature such as the study by (Jackson, 2022) highlights that although the Montessori approach is effective in building a foundation of understanding, the transition from concrete learning to mathematical abstraction takes longer than conventional methods, which can be an obstacle when learning time is limited. To overcome these challenges, teachers can implement strategies such as small group sharing based on students' level of understanding or providing individual assistance for students who need extra help. In addition, further enrichment of teacher training on Montessori strategies could help them optimise the use of didactic materials in a more time-efficient manner, without compromising the essence of independent exploration. Teachers could also collaborate with peers or conduct more in-depth supervision to ensure that every student can grasp the concept of fractions well without feeling rushed.

Conclusion

The conclusion of this study shows that the use of Montessori didactic materials in fraction learning is very effective in helping students understand this concept deeply. Concrete manipulative materials, such as fraction circles and fraction skittles, allow students to learn through independent exploration and visualisation of abstract concepts into something more tangible and easily understood. The results of observations and interviews with teachers show that the Montessori method has a positive impact on the development of students' problem-solving skills, where they are able to connect the concept of fractions with the relationship between numbers through a fun and interactive approach. In addition, this research emphasizes the important role of the teacher as a facilitator in supporting the learning process of fractions. The teacher helps guide students through the exploration of manipulative materials without directly giving answers, thus encouraging students to find their own understanding. This independence strengthens students' long-term understanding of fractions and increases their confidence in using the concept in more complex situations. This research shows that, despite some challenges such as variations in students' level of understanding, the Montessori approach can provide a strong foundation in the understanding of basic mathematics, particularly fractions, at the primary school level.

Refferences

Ahlquist, E.-M. T., & Gynther, P. (2020). Teaching in the Montessori Classroom: Investigating Variation Theory and Embodiment as a Foundation of Teachers' Development. Journal of Montessori Research, 6(1), 33–45.

Alshehri, S. (2017). The Comparison of Physical/Virtual Manipulative on Fifth-Grade Students' Understanding of Adding Fractions. University of Cincinnati.

Anggreini, D., & Priyojadmiko, E. (2022). Penerapan Media Inovatif dalam Meningkatkan Pembelajaran Matematika Materi Pecahan Sekolah Dasar pada Era Merdeka Belajar. Jurnal Pendidikan Dasar, 3(2), 82–93.

Asoy, E., Boston, E., Madagmit, I. M., & Bacatan, J. (2022). Manipulatives in learning fraction for improving first-year elementary students' understanding. Indonesian Journal of Teaching in Science, 2(2), 175–182.

Aziza, A., Pratiwi, H., & Koernarso, D. A. P. (2020). Pengaruh Metode Montessori dalam Meningkatkan Pemahaman Konsep Matematika Anak Usia Dini di Banjarmasin. Al-Athfal: Jurnal Pendidikan Anak, 6(1), 15–26.

Azizah, F. P. (2024). Dunia Pendidikan: Epistimologi Pendidikan Anak Usia Dini Dalam Perspektif Maria Montessori. TANJAK: Journal of Education and Teaching, 5(1), 31–45.

Beilstein, S. O. (2019). Supporting children's conceptual understanding of fractions with manipulatives and gesture. University of Illinois at Urbana-Champaign.

Bungao-Abarquez, E. (2020). The use of manipulative in teaching elementary mathematics. International Journal of Linguistics, Literature and Translation, 3(11), 18–32.

Cipta, D. A. S., Kartika, E. D., & Kurniawati, A. (2020). Pembelajaran Matematika Untuk Siswa Pervasive Developmental disorder-Not Otherwise Specified Melalui Montessori. Media Nusa Creative (MNC Publishing).

Crneckiy, A. E. (2022). Montessori early childhood teachers' preparedness to teach mathematics.

Culclasure, B. T., Daoust, C. J., Cote, S. M., & Zoll, S. (2019). Designing a logic model to inform Montessori research. Journal of Montessori Research, 5(1), 35–49.
Fauzi, I., & Arini, R. (2021). Desain didaktis penjumlahan dan pengurangan pecahan campuran di sekolah dasar. Jurnal Didaktika Pendidikan Dasar, 5(1), 1–30.
Goss, A. M. (2022). Young Children's Mathematical Spatial Reasoning in a Montessori Classroom. Université d'Ottawa/University of Ottawa.
Hallumoğlu, K. Ö., Karsak, H. G. O., & Maner, A. F. (2023). The Effect of the Montessori Method Integrated with Collaborative Learning on Early Mathematical Reasoning Skills. International Journal of Contemporary Educational Research, 10(4), 917–929.
Hamsa, H. A. (2020). Implementasi Metode Montessori Dalam Membentuk Arakter Kemandirian Pada Anak Usia Dini Di Brainy Bunch International Islamic Montessori School Malaysia.
Hanifah, N., Magfiroh, N. H., & Assa'diy, A. A. (2024). Analisa Efektivitas Metode Montessori terhadap Kemampuan Atensi Anak ADHD. Aulad: Journal on Early Childhood, 7(2), 434–444.
Isnawan, M. G. (2023). Didactical Design Research (I. K. Sukarna, Ed.; 1st ed.). Nashir Al-Kutub Indonesia.
Isnawan, Muhamad G, Suryadi, D., & Turmudi, T. (2022). Barriers to student learning in operating fractions. AIP Conference Proceedings, 2566(1).
Jackson, J. R. (2022). Maintaining the montessori method in louisiana public schools: a qualitative descriptive study. Grand Canyon University.
Kofa, L. (2017). Elementary teachers' perceptions of mathematics instruction in Montessori and traditional classrooms.
Lillard, A. S. (2017). Montessori: The science behind the genius. Oxford University Press.
Mavric, M. (2020). The Montessori Approach as a Model of Personalized Instruction. Journal of Montessori Research, 6(2), 13–25.
Meiana, P. U. (2023). Manajemen Kurikulum Pendidikan Montessori Dalam Perspektif Merdeka Belajar. UIN Prof. KH Saifudidin Zuhri.
Monzon, E. (2020). A Whole-Making Fractions Tangible. Diploma Work.
Okoth, O. P. (2021). Exploring the use of concrete materials in teaching and learning of fractions in grade three: a case study of a public primary school in Homa-bay County, Kenya.
Parungao, R. C. (2021). Manipulative Approach in Teaching Fractions. International Journal of Multidisciplinary: Applied Business and Education Research, 2(5), 435–447.
Phillips, B. (2022). The Montessori Method and the Neurosequential Model in Education (NME): A Comparative Study. Journal of Montessori Research, 8(2), 33–43.
Pouw, W. (2017). The cognitive function of manual activity in learning and problem solving.
Rosanah, E. (2022). Pengaruh Penggunaan Media Puzzle Berbasis Metode Montessori untuk Meningkatkan Kemampuan Pemecahan Masalah pada Anak Usia 4 tahun. Jaktarta: Fakultas Ilmu Tarbiyah dan Keguruan UIN Syarif Hidayatullah Jakarta.
Saylor, L. L., McKenzine, G. K., & Sacco, C. C. (2018). Teacher-Centered Mentorship as Meaningful Professional Montessori Development. Journal of Montessori Research, 4(2), 10–32.
Suryadi, D. (2019). Penelitian desain didaktis (DDR) dan implementasinya. Gapura Press Bandung.
Tamara, R. (2022). AZ Tanya Jawab Montessori & Parenting. Bentang Pustaka.
Ummah, S. K. (2021). Media Pembelajaran Matematika (Vol. 1). UMMPress.
Utami, M. P. (2023). Manajemen kurikulum pendidikan Montessori dalam perspektif Merdeka Belajar. Universitas Islam Negeri Saifuddin Zuhri (Indonesia).
Vivant, S. (2022). Investigating if Montessori Education Creates Deep Learning Ecologies in the Classroom: A Case Study. Northcentral University.
Wilkie, K. J., & Roche, A. (2023). Primary teachers' preferred fraction models and manipulatives for solving fraction tasks and for teaching. Journal of Mathematics Teacher Education, 26(6), 703–733.
Yayuk, E. (2019). Pembelajaran Matematika Sekolah Dasar (Vol. 1). UMMPress.